Title: If Data Could Talk – "What Regression Model Would The Statistics Represent?"

Brief Overview:

The students will investigate and create data to analyze with the TI-83 plus calculator. Students will use real-world data to discover what regression model will best fit the data. The unit will proceed from the linear regression, to the exponential regression, and then on to the logistic.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*

- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*

- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*

- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*

- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

  <u>Algebra</u>
  Students will understand patterns, relations, and functions. They will analyze and interpret various algebraic models and use appropriate algebraic equations. Students will move smoothly from the verbal, numeric, symbolic, and graphic representations of functions.

  <u>Data Analysis and Probability</u>
  Students will input data into the lists in a TI-83 plus graphing calculator and/or mini-tab in order to display their data on scatter plots. They will determine regression equations that model the data using the TI-83 plus graphing calculator and/or mini-tab. The students will also make inferences and predictions that are based on the data.

- **Process Standards**

  Problem Solving
  Students will build new mathematical knowledge through problem solving. Problems will range from sports to biology.

  Communication
  Students will communicate the knowledge they have gained verbally, numerically, graphically, and symbolically.

  Connections
  Students will connect these ideas to other real life situations that perhaps were not addressed in this lesson.

  Representation
  Students will represent the data by organizing, displaying, analyzing, and interpreting it.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- **1.1.1**
  The student will recognize, describe and extend patterns and functional relationships that are expressed numerically and algebraically.

- **1.1.2**
  The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.2.1**
  The student will determine the equation for a line.

Data Analysis and Probability

- **3.1.1**
  The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.1.2**
  The student will use the measures of central tendency and/or variability to make informed conclusions.

- **3.1.3**
  The student will calculate theoretical probability or use simulations or statistical inferences from data to estimate the probability of an event.

- **3.2.1**
  The student will make informed decisions and predictions based upon the results of simulations and data from research.

- **3.2.2**
  The students will interpret data and/or make predictions by finding and using a line of best fit and by using a given curve of best fit.

- **3.2.3**
  The student will communicate the use and misuse of statistics.

Grade/Level:

Grades 9-12, Concepts of Algebra, Algebra I & II, Pre-Calculus, A.P. Statistics

Duration/Length:

Two to three 45 minute class periods. On a block schedule, one and a half classes. The first part can take a period and the second part would take another period on a 45 minute schedule. The subject being taught will be an indicator as to how much or how little will be covered. Typically a Concepts of Algebra, and an Algebra I class would want to go through part 1 and part 2. An Algebra II, Pre-Calculus, and A.P. Statistics class might proceed from part 2 and go right on to the extensions with exponential regression and logistic regression.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Input data in lists in the TI-83/TI-83+ graphing calculator.
- Construct scatter plots in the TI-83/ TI-83+ graphing calculator.
- Create and understand a line of "best fit".
- Be familiar with spread sheets.
- Understand rate of change.
- Understand *y*-intercept.
- Round numbers appropriately.
- Set appropriate parameters for graphs.
- Plot points on graphs.
- Determine slope from linear models.
- Write equations from linear data (slope and points).

Student Outcomes:

Students will:

- collect data
- find the five number summary for a given set of data
- create scatter plots
- create a modeling line
- create a line of best fit with the TI-83/TI-83+ graphing calculator
- create a linear regression model with the TI-83/TI-83+ graphing calculator
- create an exponential regression model with the TI-83/TI-83+ graphing calculator
- create a logistic regression model with the TI-83/TI-83+ graphing calculator
- make predictions about data
- define and understand correlation coefficient

Materials/Resources/Printed Materials:

- TI-83 or TI-83+ graphing calculator
- Mini – tab (optional)
- Computer (optional)
- Student activity sheets, homework sheets, and assessment sheets
- Box of spaghetti
- Graph paper

Development/Procedures:

The teacher will introduce the lesson by going over with their students how to collect data in lists, and display data in a scatter plot on the TI-83 + graphing calculator. The students will be in cooperative learning groups to generate their own data. They will proceed from the modeling line with the use of spaghetti to the linear regression line. The students will discover that the mean point of the data they have is always on the linear regression line. They will have an opportunity to continue to explore using the exponential model with the use of the worksheet(s) and teacher guidance. There is an opportunity to continue on to the logistic model.

References: *The World Almanac and Book of Facts 2002*, *pp*. 910, 927

Assessment:

Assessment is on going through out this lesson. The teacher will be communicating with students informally through question and answer responses, gathering results from cooperative learning groups, homework, and a formal BCR assessment quiz or part of a unit test.

Extension/Follow Up:

- Students will have an opportunity with the aid of a teacher to explore exponential regression.
- Students will have an opportunity with the aid of a teacher to explore logistic regression.

Authors:

Pete Anderson
Mergenthaler Voc Tech High School
Baltimore, Maryland

Marcia J. Gampel
North County High School
Anne Arundel County, Maryland

Larry Overton
Theodore Roosevelt Senior High School
Washington, DC

Teacher Notes

This section of the unit will allow your students to plot data and then use it to determine facts or make predictions. This exercise can be done manually or with the use of technology. Allow students to work in small groups or independently to promote comfort with the material. Large grid paper, tag-board or construction paper might be helpful in demonstrating to groups of students.

Prerequisite Knowledge:

- Rounding
- Setting appropriate parameters for graphs
- Plotting graphs
- Determining slope from linear models
- Writing equations from linear data (slope and points)

Student Outcomes:

Students will be able to:

- write a linear equation from data in a scatter plot
- use technology to find a linear regression from data
- extract information or make predictions from data

Materials/Resources/Printed Materials:

- Warm-up
- Graph Paper
- Box Spaghetti
- TI-83 or TI-83 Plus Graphics Calculator/Overhead
- Worksheet for TI-83

Development/Procedures:

Day 1

1. As students enter the class, have them place a sticky dot on the Height/Shoe Size Chart corresponding to their height and shoe size (See Attachment). This information can be used as an assessment, an individual practice, or as a homework assignment.
2. Have students complete the Cars Registered in the U.S. Handout. Review answers. This handout is the basis for this Linear Regression Lesson.

3. This lesson may be done in small groups or individually. You may choose to omit the manual plotting if your students are technology fluent.
4. Review procedures for constructing a scatter plot and determining the parameters.
5. Have students to complete the scatter plot.
6. Review procedures for determining a *line of best fit* and *writing an equation from raw data*.
7. Distribute spaghetti to students for use in finding the *line of best fit*.
8. Have students complete assignment, finding the line of best fit and the equation. Incorporate the use of the median of the '*x*' and '*y*' values as needed.
9. Redo the lesson using technology, if desired, following the handout.
10. Have students compute the *five number summary* for the data (minimum, maximum, 1st quartile value, 3rd quartile value, and median).

TEACHER PREPARATION PRIOR TO LESSON:

Student Height vs Weight Profile

Initial Activity
Day 1

<u>Directions to teacher</u>:

This chart can be created on Large Grid Paper, Large Tag-Board or Construction Paper. Post it in a visible place and provide colored sticky dots for marking height vs. shoe sizes. Use a scale that is representative of the heights of your students.

Remind the females to convert their shoe sizes to male sizes or create two charts or color code dots M/F (male and female). The data can be used for assessment, home assignment, or as an extension.

HEIGHT															
	78"														
	72"														
	66"														
	60"														
	54"														
	48"														
		6	6½	7	7½	8	8½	9	9½	10	10½	11	11½	12	12½

SHOE SIZE

Lesson 1: Part 1

As students enter the class, instruct them to place a sticky dot on the chart indicating their height relative to their shoe size (This information may be organized later so that the students can interpret and make predictions). Dots will be scattered to represent the individual height-shoe size relationships…note* have girls convert their sizes to male equivalent or plot separate charts.

1. As students complete the scatter plot distribute the warm up “Cars Registered in the U.S.”.
2. Students will complete with necessary assistance of teacher.
3. Technology portion may be used or omitted as per level of students.
4. Use spot-checks, share or other methods to insure sufficient involvement and mastery at this level.
5. Discuss.

Student Activity

This information was obtained from the World Almanac and Book of Facts 2002. The source is the U.S. Department of Transportation, Federal Highway Administration.

1. Round the figures from the number of *Cars Registered in the U.S.* to the nearest thousand and million, and then place the answers in the table.

Cars Registered in the U.S., 1900-99

Year	Cars Registered	Cars Registered (Thousands)	Cars Registered (Millions)
1900	8,000		
1905	77,400		
1910	458,377		
1915	2,332,426		
1920	8,131,522		
1925	17,481,001		
1930	22,034,753		
1935	22,567,877		
1940	27,465,826		
1945	25,796,985		
1950	40,339,077		
1955	52,144,739		
1960	61,671,390		
1965	75,257,588		
1970	89,243,557		
1975	106,705,934		
1980	121,600,843		
1985	127,885,193		
1990	133,700,497		
1995	128,386,775		

2. Select an adequate scale and plot the data from ‘cars registered per million’ as a scatter plot. ‘X’ scale can be ’00 ’05 ’10 ’15 … where ’00 represents 1900. 110 will represent 2010.
3. Use a strand of spaghetti to find a line of best fit for the data. Try to include the median of both your “*x*” and “*y*” values.
4. Find the slope of your line of best fit.
5. Write an equation to fit your line.
6. Use your line to predict the number of cars that might be registered in the year 2010.
7. Write a short response as to the constraints that might prevent the *number of cars registered* from continuing to increase.

Teacher Answer Key
Cars Registered in the U.S., 1900-99

Year	Cars Registered	Cars Registered (Thousands)	Cars Registered (Millions)
1900	8,000	**8**	**0**
1905	77,400	**77**	**0**
1910	458,377	**458**	**0**
1915	2,332,426	**2,332**	**2**
1920	8,131,522	**8,132**	**8**
1925	17,481,001	**17,481**	**17**
1930	22,034,753	**22,035**	**22**
1935	22,567,877	**22,568**	**23**
1940	27,465,826	**27,466**	**27**
1945	25,796,985	**25,797**	**26**
1950	40,339,077	**40,339**	**40**
1955	52,144,739	**52,145**	**52**
1960	61,671,390	**61,671**	**62**
1965	75,257,588	**75,258**	**75**
1970	89,243,557	**89,244**	**89**
1975	106,705,934	**106,706**	**107**
1980	121,600,843	**121,601**	**122**
1985	127,885,193	**127,885**	**128**
1990	133,700,497	**133,700**	**134**
1995	128,386,775	**128,387**	**128**

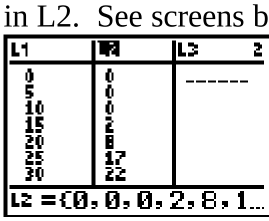
Calculator-Based Activities: The TI 83 can be used to do linear regression in the following manner:

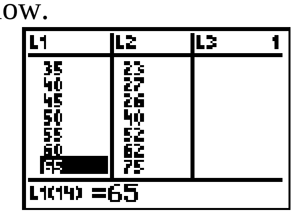
Cars Registered in the US (1900-99)

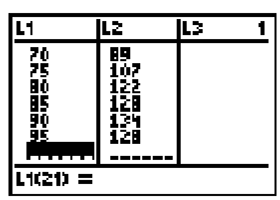
Year	Cars Registered	Cars Registered (Thousands)	Cars Registered (Millions)
1900	8,000	8	0
1905	77,400	77	0
1910	458,377	458	0
1915	2,332,426	2,332	2
1920	8,131,522	8,132	8
1925	17,481,001	17,481	17
1930	22,034,753	22,035	22
1935	22,567,877	22,568	23
1940	27,465,826	27,466	27
1945	25,796,985	25,797	26
1950	40,339,077	40,339	40
1955	52,144,739	52,145	52
1960	61,671,390	61,671	62
1965	75,257,588	75,258	75
1970	89,243,557	89,244	89
1975	106,705,934	106,706	107
1980	121,600,843	121,601	122
1985	127,885,193	127,885	128
1990	133,700,497	133,700	134
1995	128,386,775	128,387	128

A. To Make a Scatterplot and do a Linear Regression:

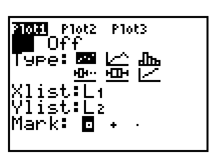
Step 1: Press STAT/1:Edit. Enter the year (last 2 digits) in L1 and number of cars registered in L2. See screens below.

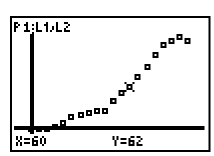






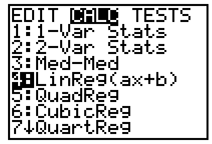
Step 2: Press 2nd/STAT PLOT/1:Plot1. Set values to the first screen at right and press ZOOM/9:ZoomStat. This gives the second screen at right.

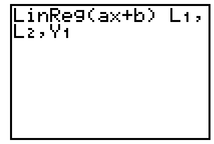


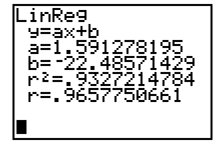


[Continued on the next page]

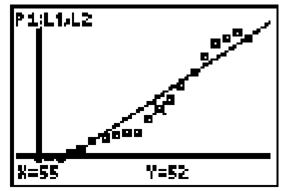
Step 3: Go to Home Screen (2^{nd}/Quit). We will now do a Linear Regression.
Step 4: Press STAT/CALC/4:LinReg(ax+b).
See first screen at right.
Press ENTER. Press L1(2^{nd}/1), L2(2^{nd}/2), VARS/Y-VARS/1:Function/1:Y1.
This gives the third screen at right.







Step 5: To see a graph of the regression line superimposed on the scatterplot, Press ZOOM/9:ZoomStat. This gives the screen at right. (See lesson 2 page 4 for directions.)



Follow-up:

Have the students organize the shoe size vs. height data that was obtained on the large tag-board. Have them to organize it by shoe size from smallest to largest. Remind them to convert girl sizes to the equivalent boy size… or create two separate charts (boys/girls).

The students will use this information to make a scatterplot and do a linear regression.

After completing the task provide the students with heights and shoe sizes of people outside the class (celebrities, athletes, etc.) in order to make the exercise more meaningful. For example, Shaquille O’Neal, height 7’1’’, shoe size 21.

Have the students compare the actual data with the model. A writing assignment can be used to summarize the findings.

If Data Could Talk: Lesson 1 Part 2
Teacher's Guide

Objectives: The students will be able to:

- use the TI-83/TI-83 + calculator to display scatter plots of linear functions.
- use the TI-83/TI-83 + calculator to find the linear regression equation.
- use the linear regression equation to make predictions.
- decide whether those predictions are over or underestimates of the actual values.
- determine the correlation coefficient and understand its importance in how well a line is determined by that number.

Materials: Student Activity Worksheet
TI-83/TI-83 + Calculator
Tape measures with cm units

Procedure: A brief introduction to this lesson should include a review of importing data in lists in the TI-83/TI-83 + Calculator. The teacher could have the students input data in a list as a warm-up since this is a review of something they have already done. They can do the scatter plot and carry as far as they can go before the teacher steps in. The teacher can use an example on the overhead calculator. After a class discussion on lists, getting the data displayed in a scatter plot, finding the linear regression equation, and reviewing the correlation coefficient, the class can be divided into groups. The size of the group depends on the total class size. Tape measures should be distributed. The students can decide on the division of labor according to the cooperative learning style used by the teacher. Each student, however, will be involved in measuring their own shoe length and arm span. Students they can work in pairs. The student activity sheet is included. Depending on time constraints, the teacher could extend this to have all groups report their results on the board or overhead and bring the students together to answer the questions after they had an opportunity to do them in their groups.

Assessment: The students will be assessed through a quiz. The teacher has a number of options. The assessment, included, can be used as a quiz or as part of a test since they are BCRs (brief constructed responses). Having it as part of a test, lends the teacher an opportunity to include SRs (student responses-multiple choice), SPRs (student produced responses-grid-ins), and ECRs (extended constructive responses). In addition, there is a holistic grading rubric that can be used to score students.

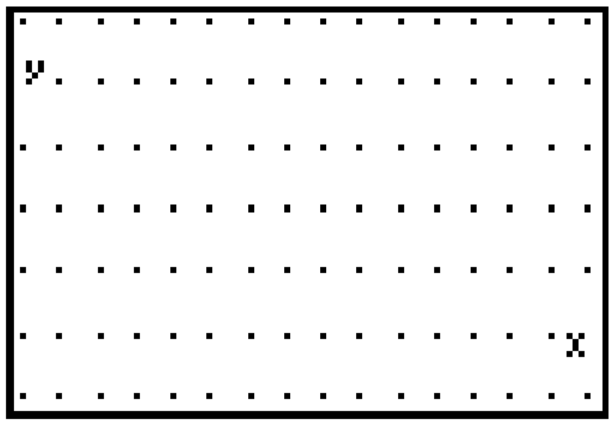
STUDENT ACTIVITY SHEET: **MEASURING ARM SPAN AND SHOE LENGTHS**

The table below shows the arm span and shoe lengths of several students.

Shoe length in cm								
Arm span in cm								

1. Let *x* represent a student's shoe length and *y* represent his arm span. Enter the data and set up a scatter plot for it on a graphing calculator. Show your WINDOW settings in the space provided and show your scatter plot on the grid.

WINDOW
Xmin =
Xmax =
Xscl =
Ymin =
Ymax =
Yscl =
Xres =



2. Describe the overall trend in the data.

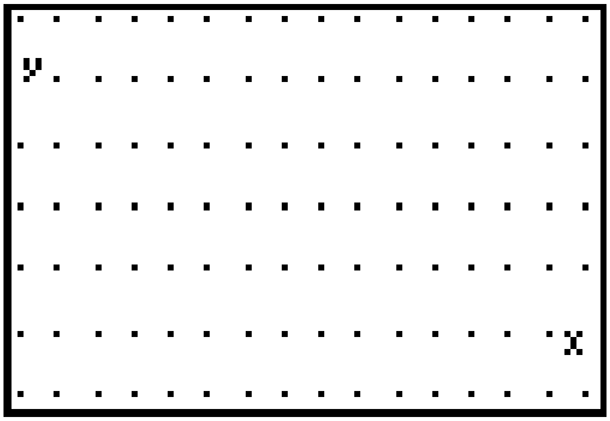
3. a. What point must lie on the graph of the linear regression model for a set of data?

   b. What are the coordinates of this point for the given data set?

   c. What does this point represent for the given situation?

4. Determine the equation of a line of best fit for the data set using the linear regression function on the calculator.

5. Graph the linear regression function along with the data on the calculator. Give the value of the correlation coefficient. Describe how well the graph of the equation fits the data, basing your discussion on what the value of the correlation coefficient means in the context of the problem



6. a. What is the slope of the line of best fit?

   b. What does the slope tell you about the relationship between shoe length and arm span?

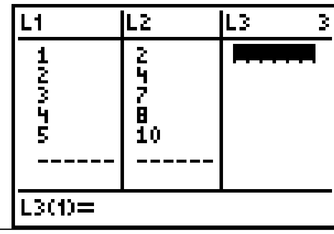
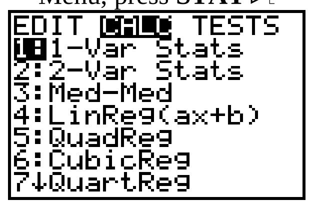
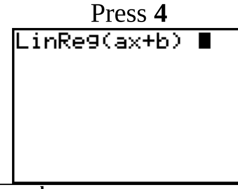
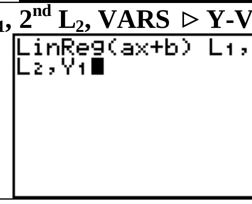
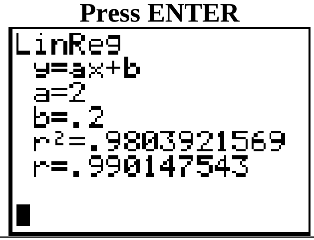
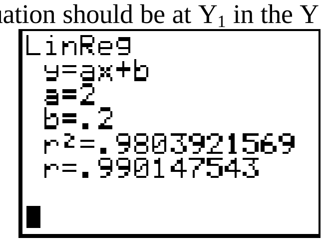
7. a. What is the actual arm span for the student whose shoe length is 24 cm?

   b. What is the predicted arm span for a student whose shoe length is 24 cm?

   c. How do these values compare? That is, does the prediction overestimate or underestimate the actual value?

8. Predict the shoe length of a student whose arm span is 42 cm using the linear regression model.

Instructions For Determining and Displaying The Linear Regression Model for a set of Data

Procedure	TI-83 + Instructions
Enter the data in L_1 and L_2. Use the data shown in the display.	To access the lists press **STAT ENTER** 
Select the **LinReg** model in the **STAT CALC** menu.	To access the **STAT CALC** Menu, press **STAT** ▷  To select the **LinReg** model Press **4** 
Enter the appropriate lists for the *x*-values and *y* – values. *****On the TI-83, also enter the location for the linear regression model equation.	**Press 2nd L_1, 2nd L_2, VARS ▷ Y-VARS ▷ 1 ▷ 1** 
Calculate the equation of the linear Regression model.	**Press ENTER** 
Display the graph of the regression equation along with the scatter plot of the data.	The equation should be at Y_1 in the Y = screen.  Set up the scatter plot, then graph by pressing **ZOOM 9**

STUDENT ASSESSMENT ON MEASURING ARM SPAN AND SHOE LENGTHS

The table below shows the arm span and shoe lengths of several students.

Shoe length in cm	23	25	29	31	32	24	28	27
Arm span in cm	31	36	47	50	49	32	44	40

1. Let *x* represent a student's shoe length and *y* represent his arm span. Enter the data and set up a scatter plot for it on a graphing calculator. Show your WINDOW settings in the space provided and show your scatter plot on the grid below at the bottom of page.

2. Describe the overall trend in the data.

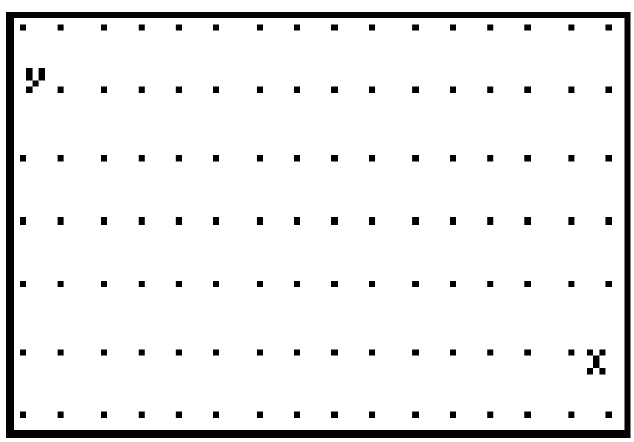
3. a. What point must lie on any graph of the linear regression model for a set of data?

   b. What are the coordinates of this point for the given data set above?

   c. What does this point represent for the given situation?

4. Determine the equation of the line of best fit for the data set using the linear regression function on the calculator.

WINDOW
Xmin =
Xmax =
Xscl =
Ymin =
Ymax =
Yscl =
Xres =



5. Using the scatter plot in question 1:
   - Graph the linear regression function along with the data on the calculator.
   - Give the value of the correlation coefficient.
   - Describe how well the graph of the equation fits the data, basing your discussion on what the value of the correlation coefficient means in the context of the problem

6. a. What is the slope of the line of best fit?

   b. What does the slope tell you about the relationship between shoe length and arm span?

7. a. What is the actual arm span for the student whose shoe length is 24 cm?

   b. What is the predicted arm span for a student whose shoe length is 24 cm?

   c. How do these values compare? That is, does the prediction overestimate or underestimate the actual value)?

8. Predict the shoe length of a student whose arm span is 42 cm using the linear regression model.

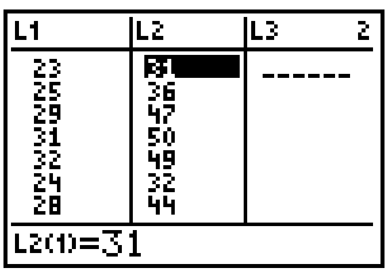
STUDENT ASSESSMENT ON MEASURING ARM SPAN AND SHOE LENGTHS

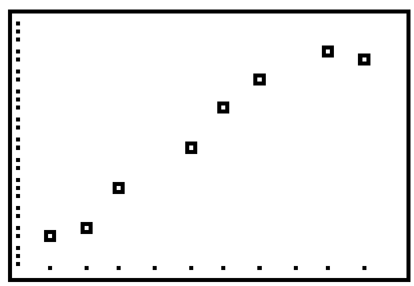
Answer Key

The table below shows the arm span and shoe lengths of several students.

Shoe length in cm	23	25	29	31	32	24	28	27
Arm span in cm	31	36	47	50	49	32	44	40

1. Let *x* represent a student's shoe length and *y* represent his arm span. Enter the data and set up a scatter plot for it on a graphing calculator. Show your WINDOW settings in the space provided and show your scatter plot on the grid at the bottom of the page. **Answers can vary as long as the minimum *x* (*y*) is a little lower than the lowest shoe length (arm span) and the maximum *x*(*y*) is a little higher than the greatest shoe length (arm span). You can just have them hit ZOOM 9 and let the calculator do it.**





2. Describe the overall trend in the data. **The overall trend in the data is positive.**

3. a. What point must lie on the graph of the linear regression model for a set of data? **The mean *x* and the mean *y* must lie on the graph of the linear regression model for a set of data.**

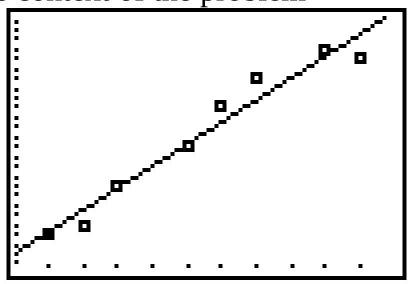
   b. What are the coordinates of this point for the given data set? **The coordinates of this point are $x = 27.375$ and $y = 41.125$**

   c. What does this point represent for the given situation? **This point represents the average shoe length and arm span in cm.**

4. Determine the equation of a line of best fit for the data set using the linear regression function on the calculator. **The equation of best fit for the data set using the linear regression function on the calculator is**

$$y = 2.2690355329949x - 20.98984771535$$

5. Graph the linear regression function along with the data on the calculator. Give the value of the correlation coefficient. Describe how well the graph of the equation fits the data, basing your discussion on what the value of the correlation coefficient means in the context of the problem



The correlation coefficient is .9789560002. The graph of the equation fits the data quite well because this correlation coefficient lies between the interval of –1 to 1, which indicates a good line. The closer to 1 or -1 the better the fit. Our .9789560002 is close to 1 telling us also our rate of change is positive.

6. a. What is the slope of the line of best fit? **The slope of the line of best fit is 2.269.**

   b. What does the slope tell you about the relationship between shoe length and arm span? **The slope tells us that for approximately every 2.269 cm of arm span a student's shoe length is 1 cm long.**

7. a. What is the actual arm span for the student whose shoe length is 24 cm? **The actual arm span for the student whose shoe length is 24 cm is 32 cm.**

   b. What is the predicted arm span for a student whose shoe length is 24 cm? **The predicted arm span for a student whose shoe length is 24 cm is 33.467 cm.**

   c. How do these values compare? That is, does the prediction overestimate or underestimate the actual value)? **The prediction is an overestimate of the actual value.**

8. Predict the shoe length of a student whose arm span is 42 cm using the linear regression model. **The shoe length of a student whose arm span is 42 cm using the linear regression model is 27.76 cm.**

HOLISTIC GRADING RUBRIC FOR ASSESSMENT ON MEASURING ARM SPAN AND SHOE LENGTHS

Question 1

- **Entering data in lists**
- **Window**
- **Scatter plot**

Score

- **All points covered correctly will receive a grade of *E*ssential.**
- **Any two points covered correctly will receive a grade of *P*artial.**
- **Only one point or no point covered will receive a grade of *I*ncomplete.**

Question 2

- **Only acceptable answer is positive**

Score

- ***E*ssential, if the above answer is there or *I*ncomplete, if the above answer is not.**

Question 3

- **Actually stating that it is the mean point**
- **The actually mean point, with correct coordinates, for this data**
- **What this point means in the context of this problem**

Score

- **All points covered correctly will receive a grade of *E*ssential.**
- **Any two points covered correctly will receive a grade of *P*artial.**
- **Only one point or no point covered will receive a grade of *I*ncomplete.**

Question 4

- **Calculated answer from TI-83/TI-83 Plus**

Score

- **A grade of *E*ssential is obtained when the calculated answer is obtained.**
- **A grade of *P*artial is obtained when the answer is obtained using the student's own hand drawn modeling line (as long as a positive slope close to 2 is indicated).**
- **Anything else is an *I*ncomplete.**

Question 5

- **Scatter plot with linear regression line on it**
- **Discussion of the correlation coefficient:** $-1 \leq r \leq 1$

Score

- **A grade of *E*ssential is obtained when the linear regression line is actually seen on the scatter plot, along with a discussion of what the correlation coefficient is and how well the graph of the equation fits the data because of that number.**
- **A grade of *P*artial is obtained when only one of the above points is correctly covered.**
- **A grade of *I*ncomplete is obtained when nothing is covered correctly.**

Question 6

- **The slope is answered correctly from calculator/or modeling line**
- **The meaning of the slope in terms of the context of the problem**

Score

- **A grade of *E*ssential is obtained if both the numeric and verbal part of the answer is there.**
- **A grade of *P*artial is obtained if the word relationship is correct.**
- **A grade of *I*ncomplete is obtained if only the numeric part is mentioned.**

Question 7

- **Actual**
- **Predicted**
- **Comparison**

Score

- **All points covered correctly will receive a grade of *E*ssential.**
- **Any two points covered correctly will receive a grade of *P*artial.**
- **Only one point or no point covered will receive a grade of *I*ncomplete.**

Question 8

- **Calculated answer with the correct linear regression equation/answer obtained by student's equation or obtained by hand**

Score

- **A grade of *E*ssential is obtained when the student has given the answer by the correct linear regression line or by their hand drawn modeling line.**
- **A grade of *I*ncomplete is obtained if the answer is incorrect according to the calculated linear regression line or the student's modeling line.**

This quiz is based on 24 points. Each question is worth 3 points. Questions 7 and 8 with just a possible score of *E*ssential or *I*ncomplete would either get a 3 or a 0 respectively. Questions 1, 3 – 7 have possible scores of *E*ssential, *P*artial, or *I*ncomplete will receive 3, 2 or 1, or 0 respectively. A note on the *P*artial score receiving either a 2 or 1 is based on the teacher taking a step back and looking at that question "holistically". That is to say, intuitively, getting a feel as to whether the response is *developing* (for a score of 2) or *minimal* (for a score of 1).

A teacher could use this as part of a test. They would already have 24 points to the exam, and can build there SRs, SPRs, and ECRs around this. SRs and SPRs holistically receive scores of 1 and ECRs holistically receive scores of 4.

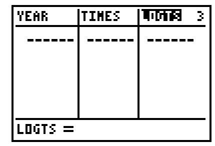
Extension: Lesson 1, Part 2
Decreasing Example : Mileage Records – 1875 to 1999

Year	Name,Country	Time in min	Time in sec	Log (time in sec)		Year	Name,Country	Time in min	Time in sec	Log (time in sec)
1875	Walter Slade, Britain	4:24.5	264.5	2.4224256		1945	Gunder Haegg, Sweden	4:01.4	241.4	2.3827372
1880	Walter George, Britain	4:23.2	263.2	2.4202858		1954	Roger Bannister, Britain	3:59.4	239.4	2.3791241
1882	Walter George, Britain	4:21.4	261.4	2.4173055		1954	John Landry, Australia	3:58	238	2.3765769
1882	Walter George, Britain	4:19.4	259.4	2.4139699		1957	Derek Ibbotson, Britain	3:57.2	237.2	2.3751146
1884	Walter George, Britain	4:18.4	258.4	2.4122925		1958	Herb Elliot, Australia	3:54.5	234.5	2.3701428
1894	Fred Bacon, Scotland	4:18.2	258.2	2.4119562		1962	Peter Snell, New Zealand	3:54.5	234.5	2.3701428
1894	Fred Bacon, Scotland	4:17	257	2.4099331		1964	Peter Snell, New Zealand	3:54.1	234.1	2.3694014
1911	Thomas Connett, U.S.	4:15.6	255.6	2.4075608		1965	Michel Jazy, France	3:53.6	233.6	2.3684728
1911	John Paul Jones, U.S.	4:15.4	255.4	2.4072208		1966	Jim Ryan, U.S.	3:51.1	231.1	2.3637999
1913	John Paul Jones,U.S.	4:14.6	254.6	2.4058583		1967	Jim Ryan, U.S.	3:51.1	231.1	2.3637999
1915	Norman Taber, U.S.	4:12.6	252.6	2.4024333		1975	Filbert Bayi Tanzania	3:51.0	231	2.3636119
1923	Paavo Nurmi, Finland	4:10.4	250.4	2.3986343		1975	John Walker, New Zealand	3:49.4	229.4	2.3605934
1931	JulesLadoumegue, France	4:09.2	249.2	2.3965480		1979	Sebastian Coe	3:49	229	2.3598355
1933	Jack Lovelace,New Zealand	4:07.6	247.6	2.3937506		1980	Steve Ovett	3:48.8	228.8	2.3594560
1934	Glenn Cunningham, U.S.	4:06.8	246.8	2.3923451		1981	Sebastian Coe	3:48.53	228.53	2.3589432
1937	Sydney Wooderson, Britain	4:06.4	246.4	2.3916407		1981	Steve Ovett	3:48.40	228.4	2.3586961
1942	Gunder Haegg, Sweden	4:06.2	246.2	2.3912880		1981	Sebastian Coe	3:47.33	227.33	2.3566568
1942	Arne Andersson, Sweden	4:06.2	246.2	2.3912880		1985	Steve Cram	3:46.32	226.32	2.3547229
1942	Gunder Haegg, Sweden	4:04.6	244.6	2.3884564		1993	Noureddine Morceli	3:44.39	224.39	2.3510035
1943	Arne Andersson,Sweden	4:02.6	242.6	2.3848908		1999	Hicham el Guerrouj	3:43.13	223.13	2.3485580
1944	Arne Andersson, Sweden	4:01.6	241.6	2.3830969						

Extension: Lesson 1, Part 2 continued
Teacher's Directions For Using TI-83 with
Decreasing Exanple : Mileage Records – 1875 to 1999

A. Creating Lists with the SetUpEditor:

The SetUpEditor is used to create "named" lists. In the example below, we will create lists – "YEAR," "TIMES," and "LOGTS."



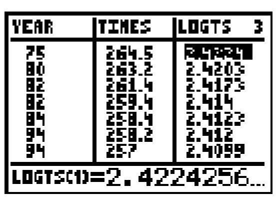
Step 1: Press STAT/5:SetUpEditor. The SetUpEditor is now ready for you to name the lists that you will use.
Step 2: Press STAT/1:Edit to see the list you created. To create the next list, HIGHLITE the title of the next column and enter the title "TIMES" and "LOGTS." This gives the second screen at right.
Step 3: Enter the times from the Table above. Be sure that when you enter the values for LOGTS, you actually take the Log (TIMES)[See Screen 1 below].

NOTE: You will only use LOGTS if you'll do a linear regression with YEAR vs. LOGTS. Otherwise you will use only the first two columns.

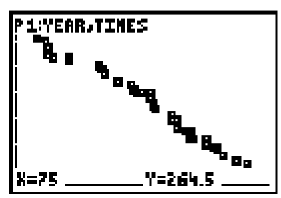
B. Creating Scatterplots:

Step 4: Press 2^{nd}/STATPLOT/1:Plot1. Set this screen to the highlited options in the screen at right. To find a list of LISTS, press 2^{nd}/LIST and select the names of the lists with which you're working.
Step 5: If you press ZOOM/9:ZoomStat, the calculator will choose the screen appropriate for your data. This plot will result in Screen 2 below.

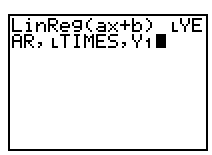
Screen 1

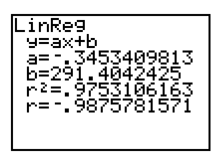


Screen 2

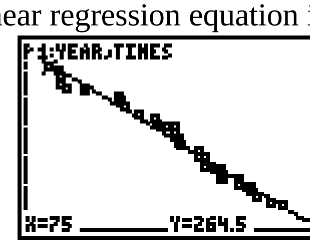


C. The Linear Regression:





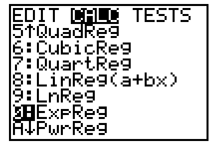
Step 6: PRESS STAT/CALC/4:LinReg(ax+b). Enter the lists entitled YEAR and TIMES, separated by commas. At the end of this you will need to enter the place you will store the equation when it's found. PRESS VARS/Y-VARS/1:Function/1:Y_1. This gives the first screen at right.
Step 7: PRESS ENTER to get the second screen above. The linear regression equation is **y = -.34534x + 291.40424.** The calculator has found the regression equation and stored it in Y_1. To see a graph of the scatterplot AND the regression equation, press

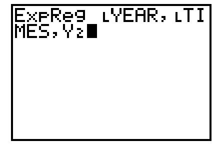


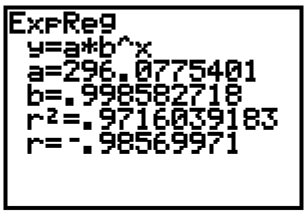
ZOOM/9:ZoomStat/ENTER. This gives the screen at right.

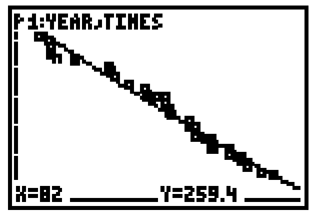
<u>D. The Exponential Regression:</u> We will use the Exponential Regression capabilities to write a different equation (maybe a better model) of the data relating YEAR to TIMES.

<u>Step 8</u>: PRESS STAT/CALC/0:ExpReg (See screen at right). Type in the remainder of the commands as seen in the second screen and press ENTER. The regression equation is as pictured below. Pressing ZOOM/9:ZoomStat gives the screen below, right.







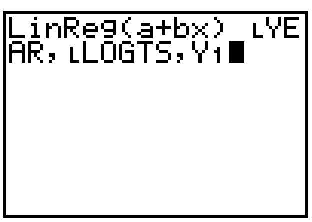


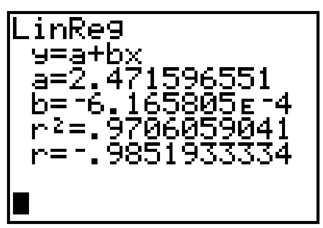
If (x , Log y) produces a linear pattern, exponential regression is appropriate because of the following argument:

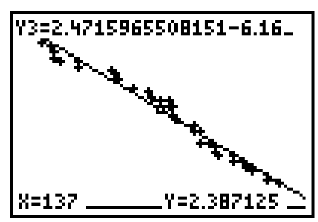
$$\mathbf{Log\ y = a + bx}$$
$$\mathbf{y = 10^{a+bx}}$$
$$\mathbf{y = 10^{a} * 10^{bx}}$$
$$\mathbf{y = 10^{a} * \left(10^{b}\right)^{x}}$$

This last line is an exponential form. An exponential regression may be performed using YEAR and LOGTS as seen in the screens below.

-







Extension: Student Worksheet

Questions on Decreasing Exanple : Mileage Records – 1875 to 1999

1. Use the linear equation and the exponential equations to predict what the record will be in 2010.

2. Show the limitation of fitting this data with the linear equation and even with the exponential model by predicting what the record will be in the year 2699 with each model.

3. According to the exponential model, how many years will it take to reduce the time to run the mile to one half of the 1875 record and what is the approximate rate of decline in seconds per year?

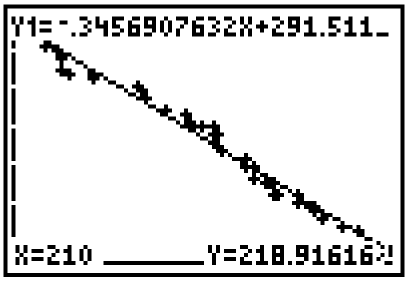
<u>**Extension: Teacher's Key to Student Worksheet**</u>
<u>**Questions on Decreasing Example : Mileage Records – 1875 to 1999**</u>

1. Use the linear equation and the exponential equations to predict what the record will be in 2010.

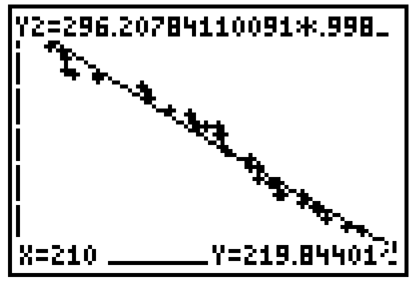
Answer (using year 1800 as 0):

Linear | Exponential

Plots Need



not new

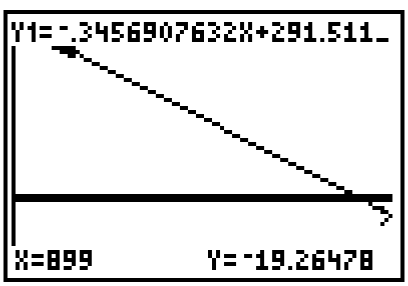


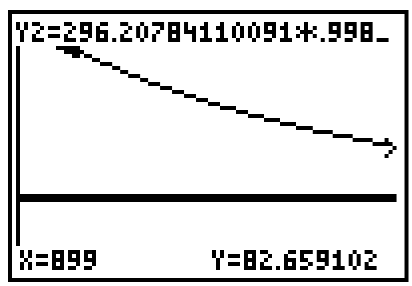
corrected. graphs.

2. Show the limitation of fitting this data with the linear equation and even with the exponential model by predicting what the record will be in the year 2699 with each model.

Answer (using year 1800 as 0):

Linear | Exponential





No one can run one mile in a negative amount of time.

1 mile/(82.659102/3600)= 43.55mph which is unlikely without mechanical or biomechanical aides even in the future, but it may be at least possible.

3. According to the exponential model, how many years will it take to reduce the time to run the mile to one half of the 1875 record and what is the approximate rate of decline in seconds per year?

Answer: One half of the 1875 time in seconds is 132.5 seconds which according to the table on the TI-83 takes 568 years from 1800 and from 1875 that is 493 years (the year 2368). The approximate half-life formula solved for P% time period is P = 70/*T* where *T* is half-life. Thus, 70/493 = .14%.

Extension: Lesson 1, Part 2 continued
Decreasing Example : Personal Weight Lifting

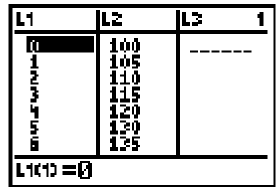
Week	Bench Press in Lb. (8 Reps, 3 sets, once/week)
0	100
1	105
2	110
3	115
4	120
5	130
6	135
7	140
8	150
9	155
10	165
11	170
12	180
13	190
14	200
15	210
16	210
17	210 no lifting this week
18	210 no lifting this week
19	210 no lifting this week
20	210
21	215
22	220
23	220
24	225
25	230

Caution: These gains may not be typical. This is for a 200 lb man who has some weightlifting experience.

Extension: Lesson 1, Part 2 continued
Teacher's Directions For Using TI-83 with
Increasing Example : Personal Weight Lifting

Creating Lists with the SetUpEditor:

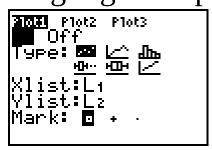
The SetUpEditor is used to create "named" lists. In the example below, we will use just the given lists – L1 for weeks and L2 for weight

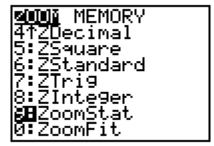


Step 1: Enter the weeks and weights from the Table above.

A. Creating Scatterplots:

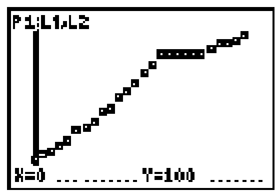
Step 2: Press 2nd/STATPLOT/1:Plot2. Set this screen to the highlighted options in the screen at right.





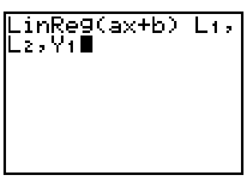
Step 3: If you press ZOOM/9:ZoomStat, the calculator will choose the screen appropriate for your data. This plot will result in Screen 2 below.

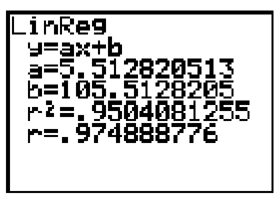
Screen 2



B. The Linear Regression:

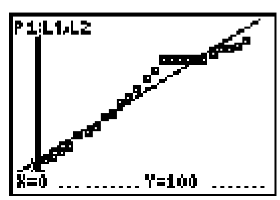
Step 4: PRESS STAT/CALC/4:LinReg(ax+b). Enter the lists L1 and L2, separated by commas. At the end of this you will need to enter the place you will store the equation when it's found. PRESS VARS/Y-VARS/1:Function/1:Y_1. This gives the first screen at right.



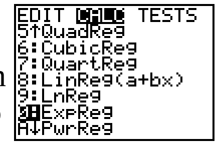


Step 5: PRESS ENTER to get the second screen above. The linear regression equation is **y = 5.512820513x + 105.5128205.** The calculator has found the regression equation and stored it in Y_1. To see a graph of the scatterplot AND the regression equation, press

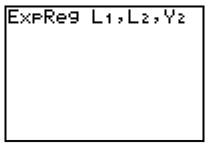
ZOOM/9:ZoomStat/ENTER. This gives the screen at right.

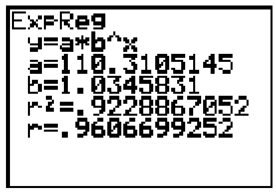


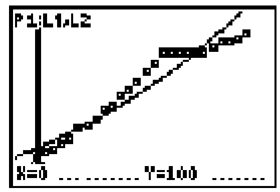
C. The Exponential Regression: We will use the Exponential Regression capabilities to write a different equation (maybe a better model) of the data relating to weeks and weight.



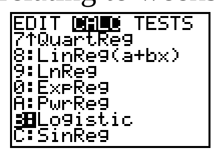
Step 6: PRESS STAT/CALC/0:ExpReg (See screen at right). Type in the remainder of the commands as seen in the second screen and press ENTER. The regression equation is as pictured below. Remember to press y =, left arrow, and ENTER to turn off y1. Pressing ZOOM/9:ZoomStat gives the screen below, right.



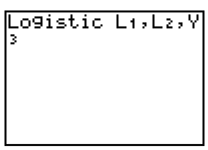


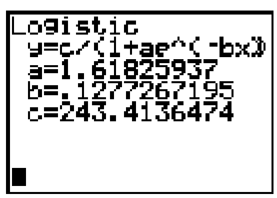


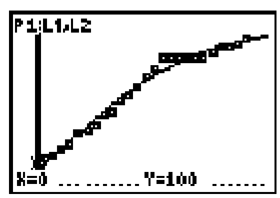
D. The Logistic Regression: We will use the Logistic Regression capabilities to write a different equation (maybe a better model) of the data relating to weeks and weight.



Step 7: PRESS STAT/CALC/B:LogReg (See screen at right). Type in the remainder of the commands as seen in the second screen and press ENTER. The regression equation is as pictured below. Remember to press y =, ENTER,left arrow, and ENTER to turn off y2. Pressing ZOOM/9:ZoomStat gives the screen below, right.







Extension: Lesson 1, Part 2 continued
Student Worksheet with
Increasing Example : Personal Weight Lifting

1. **What is the rate of increase each week and what is the doubling time for the first 15 weeks in the exponential model ?**

2. **Which model, the linear, the exponential, or the logistic, fit the entire data the best? Explain using words, symbols, or both.**

3. **Graph the data as a stat plot, give both the equations of a best fit line and a best fit exponential, and select one future year along each equation and state when and how many homeruns the next homerun record will be according to those models (you should round both the year and the number of homeruns to the nearest whole number)**

Baseball Player	Year	Homerun record
Barry Bonds	2001	73
Mark McGwire	1998	70
Roger Maris	1961	61
Babe Ruth	1927	60

Extension: Lesson 1, Part 2 continued
Teacher's Key For Student Worksheet with
Increasing Example : Personal Weight Lifting

1. **What is the rate of increase each week and what is the doubling time for the first 15 weeks in the exponential model ?**

Answer:
For 100 to 210 pounds the rate of increase is approximately 5% rounded to the nearest percent. That is

$$210 = 100 * (1+r)^{15}$$
$$2.1 = (1+r)^{15}$$
$$1.02 = 1+r$$
$$.05 = r$$

The doubling time using the approximate doubling time formula is 70/5 = 14 years. The doubling time using the exact doubling time formula is log 2 / log (1.05) = 14.2 weeks. Both of these agree with most of the table from 100 lb to 200 lb in 14 weeks.

2. **Which model, the linear, the exponential, or the logistic, fit the entire data the best? Explain using words, symbols, or both.**

Answer: Note that the logistic equation is given without a regression correlation; however, it visually seems to fit the data better on the graph. In weight lifting as in many sports, you can quickly make progress at first, and then you hit periods with little or no improvement. Of course adequate rest is extremely important, but even then your rate of growth usually slows down and there is probably a maximum that you can accomplish. Let's say in this case this is 300 lb. This maximum is called the carrying capacity with population growth. That same idea will be used here. To compare the approximate rate of growth of 2% from

$$230 = 210 * (1+r)^5$$
$$1.095= (1+r)^5$$
$$1.02 = 1+r$$
$$.02 = r$$

to the growth rate predicted by the logistic model , you must use the initial 5% to get the base growth rate in the logistic model as follows:

$$r = \frac{\text{growth rate}}{(1 - \text{weight/carrying capacity})}$$

$$r = .05/(1-210/300)$$
$$r = .16666667$$

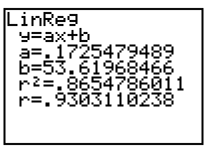
Now, you use this value of r to predict the growth rate for the weight gain.

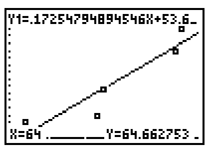
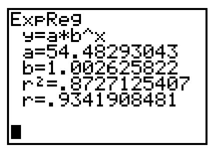
Growth rate = .16666667 * (1 – 230/300)
= .0388889

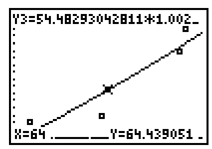
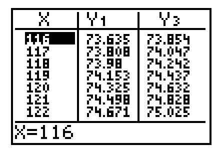
This logistic model predicts a current rate of about 3.9% which does not match the actual current growth rate of 2%. However, the rate has declined. Therefore, it is too early to tell whether the growth will continue to decline at a gradual rate as the logistic model requires.

3. Graph the data as a stat plot, give both the equations of a best fit line and a best fit exponential, and select one future year along each equation and state when and how many homeruns the next homerun record will be according to those models (you should round both the year and the number of homeruns to the nearest whole number)

Baseball Player	Year	Homerun record
Barry Bonds	2001	73
Mark McGwire	1998	70
Roger Maris	1961	61
Babe Ruth	1927	60



Possible Answers for Linear:	**2016 - 2019**	**74**
	2022 – 2026	**75**
	2027 – 2032	**76**
	2033 – 2038	**77**
Possible Answers for Exponential	**2015 - 2019**	**74**
	2019 – 2024	**75**
	2025 – 2029	**76**
	2030 – 2034	**77**